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POST-NEWTONIAN EXTENSIONS OF GRAVITY THEORY AND THE ROLE OF FUNDAMENTAL PHYSICAL CONSTANTS

Doctoral thesis resume

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Introduction

This doctoral thesis is dealing with the exceptions from the well-known law of Newtonian gravity and the conditions they appear.

A first class of exceptions appears in the non-inertial frames of reference case. The corrections to the Newtonian force are the additional terms which occur because of the movement conditions, because of the measurements conditions, independent from scale and the kind of reference frame we consider.

The second class of exceptions occurs ravitational force is subject to significant transformations, on small and large scales compared to normal scale.

The contributions of this thesis investigates a large amount of ideas, orientations, sometimes different, of the scientific research unsulved problems. The bonds of common reality are made, where there is possibility, through experimental data, or, in other cases, through well-known and consacrated theretical results.Although this thesis is a theoretical based on, it has the right to contain a small dose of scientific speculation.

This thesis is structured in conformity with a evolution of ideas. Chapter I contains the official point of view of present science, introductive elements of general relativity, with confirmations and denials of this theory. Chapter II makes the study of non-inertial frames of reference movement within a theory which is conceived itself like a generalization of the Newtonian theory of gravitation and an alternative to general relativity, at small velocities and energies. Chapter III studies the non-inertial frames of reference movement in different formalisms and practically ends the part of the thesis dedicated to non-inertial mechanics. With chapter IV begins the study of modifications which we have to make to Newtonian gravity. In this chapter I propose an alternative theory to MOND theory, the Modified Newtonian Dynamics. Chapter V builds, based on a modified Newtonian force, a cosmological model. Chapter VI represents an intermediate chapter to a multidimensional physics and exhibits some original results concerning M theory. Chapter VII shows an alternative to multidimensionability, the fractality, and demonstrates how a non-differentiable space-time can induce a modification to the Newtonian gravity force. In the final chapter, VIII, all the above results, concerning the modifications of the Newtonian gravity force, are synthesized into an original theory.

Chapter I
The place of the general relativity theory in present physics

I.1.b. Time-invariance of the fundamental physival constants

Consider two different standards, time and length. The actual standard of time is, as mentioned before, given by the frequency of the hyperfine transition of Cs atoms. In fundamental physical constants it is:

$$\upsilon_1 = \boldsymbol{m}_e \frac{\boldsymbol{m}_e}{\boldsymbol{m}_n} \alpha^2 \boldsymbol{c}^2 \frac{1}{\boldsymbol{h}} \tag{I1}$$

where the meaning of these constants is known.

Similarly, the standard of length is defined by the wavelength of the Kr-86 near to $\lambda = 605{,}78$ nm. In fundamental physical constants it is given by:

$$L_1 = \frac{1}{m_e}\frac{1}{\alpha^2}\frac{1}{c}h \qquad (I.2)$$

If we consider (I.1) and (I.2) the time and length standards then they should be constant by definition. It's hard to imagine the possibility that some fundamental constants can vary now as a function of time and length. For example, no change of the quantity:

$$\frac{L_1}{T_1} = c\alpha^2\frac{m_e}{m_n} \qquad (I.3)$$

could not be measured with our standard because we get to make some judgments that seem childish. Considering each term in the right side of equations (I.1) and (I.2) a fundamental physical constant, we could imagine in (I.3) the situation, a bit absurd, in which the expression from the left side of the equation remains constant as a result of proportional variation of constants α. and m_e / m_n. We might ask then what bizarre correlation may exist between the electron and the neutron mass ratio and fine structure constant components. We would think at some point we somehow passed the strict physics and play only with some numbers.

Next step of our approach is to stop a little at the second pair of standards of length and time necessary for the course of what follows. We are, of course, free to choose any length and time we want as standards. We will adopt the second standard of time the Compton frequency of electron, that means:

$$\upsilon_2 = m_e c^2 \frac{1}{h} \qquad (I.4)$$

Unlike (I.1) and (I.2) this expression is simpler, contains no constants in the form of reports and no fine structure constant. That is an advantage because if we choose an expression for standard of length that contains about the same constants, when we do the report of the standards of length and time we can get a simpler expression than (I.3), which can work more easily. In this vein, we consider the standard of length, the radius in units of Bohr radius of hydrogen atom:

$$L_2 = \frac{1}{m_e}\frac{1}{c}\frac{1}{\alpha}h \qquad (I.5)$$

If we ignore the standards (I.1) and (I.2) then the reference standards are expressions (I.4) and (I.5), they are constant by definition, and it would be very difficult to imagine a change in the quantity:

$$\frac{L_2}{T_2} = c\frac{1}{\alpha} \qquad (I.6)$$

to be the result of variation of fundamental physical constants from it. The situation is changing if we try to compare the standards of the same type and how they vary according to which the fundamental constants they contain. The most convenient would be to consider the simple expression:

$$\frac{L_2}{L_1} = \alpha \qquad (I.7)$$

Note that now one can easily imagine a variation of the fine structure constant, given that a standard length remains fixed reference and the other would vary inversely with the variation of the fine structure constant.

This would be the situation in the case previously considered. There might be reasoning that would contain some variation of fundamental physical constants. However, in practice, from experimental reasons we should issue a little differently. It's obvious that in our space-time frames (I.3) and (I.6) is missing something. That something is the condition of measurability, which is specific to quantum mechanics, not only to the theory of relativity. Time is measured only by space and space only by time. We can not imagine an experiment in which to measure them simultaneously. They measured each other with the condition to be known the speed, at a time. Physical time, as we know it is only a measure of movement in a certain space with a known speed. On the other side space can be measured only like duration of the movement. Therefore, because our references for space and time (I.3) and (I.6) to be operational from the experimental point of view we must to have a measurability condition, defined by:

$$\frac{nL}{pT} = c \tag{I.8}$$

where n and p positive real numbers, which are required to define multiple elementary lengths and times determined in experiments. Velocity c is known, otherwise we could make any experimental determination, is a speed limit, the speed at which light travels. Of course, the relation (I.8) must be understood only from practical reasons. A direct interpretation would lead to some absurdities: it is clear that the Bohr radius of hydrogen is not measured by the Compton electron frequency, to give just one example. In practice we only use multiples of these standards and do not care where they come from, only care to be exact. Then, a speed faster than the speed of light is specific to classical mechanics, while in the astrophysical observations is used only information that reaches us here on Earth with the speed of light, a specific speed of quantum mechanics. The speed of light must be understood only as a limitation of nature. If c could vary then we have not a space-time frame in which we can make reliable measurements.

So we have now the principles which can reconsider our problem. It is obvious that if one takes into account (I.8), in relation (I.6), since c is fixed, it can be concluded that α is a constant. Any variation of it, in a physical time frame, is null. If we consider now this very important intermediate result in relation (I.3), taking into account (I.8), it results that the electron to neutron mass ratio is a constant.

Now, if we apply a variation to the relation:

$$L_2 = \alpha L_1 \tag{I.7'}$$

and we consider that the fine structure constant variation is zero then we have:

$$\delta L_2 = \alpha \delta L_1 \tag{I.9}$$

If we apply now a variation to the relation (I.7), it will result after an elementary calculation:

$$L_1 \delta L_2 - L_2 \delta L_1 = 0 \tag{I.10}$$

From (I.9) and (I.10) follows:

$$\delta L_1 (\alpha L_1 - L_2) = 0 \tag{I.11}$$

it results the invariability of standard length (I.2). This result, which is introduced in relation (I.10), has as a consequence the invariability of standard length (I.5). And last but not least, from the invariability of standard length (I.2) can be easily deduced that the variations of the Planck constant and electron mass are simultaneously zero. Finally we conclude that the variations of all physical constants contained in (I.1), (I.2), (I.4) and (I.5) are simultaneously zero if we set the measurability condition (I.8).

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Chapter II
The movement in non-inertial reference frames

II.5 The deflection of light by the gravitational field

In this case the procedure is similar, following the same reasoning as we see in the previous case. The diferences occur because of rotation.

The starting point is the prime integrals (II.18) and (II.19). The equation (II.18) written in form:

$$\dot{\theta} = \frac{C}{r^2} - \omega$$

is multiplying with itself and it is dividing term by term with (II.19). It results::

$$\frac{d\theta}{dr} = \pm\left(\frac{C}{r^2} - \omega\right)\frac{1}{\sqrt{2h + 2\frac{\mu}{r} - C^2}}$$

$$h > 0$$

$$2h = \mu/a\,, \qquad C^2 = \mu a\left(e^2 - 1\right)$$

From the above equation it results:

$$\theta - \theta_0 = \sqrt{e^2 - 1}\int_{-\infty}^{\infty}\frac{du}{e\cosh u - 1} - \omega\Delta t$$

Taking into account the above results and that the integral is π, we have:

$$\delta\theta = -\omega\Delta t + \frac{\Delta}{2}$$

result in conformity with [2].

A different result we find if we calculate the deflection according to [1]:

$$\delta\theta = \int_{-\infty}^{\infty}\frac{\left(C - \omega r^2\right)}{\sqrt{\mu a}\left(e\cosh u - 1\right)}du - \alpha - \pi$$

Here, between constants we have the relation:

$$C = C_0 + C_1 = r^2\dot{\theta} + r^2\omega$$

meaning:

$$C - \omega r^2 = C_0$$

We have:

$$\int_{-\infty}^{\infty}\frac{C_0}{\sqrt{\mu a}\left(e\cosh u - 1\right)}du = \frac{C_0\pi}{C}$$

from which it results a more general expresion for the deflection:

$$\delta\theta = \pi\left(\frac{C_0}{C} - 1\right) + \frac{\Delta}{2} = \pi\left(\frac{C_0 - C_0 - C_1}{C_0 + C_1}\right) + \frac{\Delta}{2} = -\pi\frac{\omega}{\omega + \dot{\theta}} + \frac{\Delta}{2}$$

With the above expression we find something intriguing. If we take for ω the same value (-0”.0068/year) and for dθ/dt=359degrees59’60”/year (the movement of Earth around the Sun) we find that the contribution of the non-inertial movement is negligible. The result is also the Newtonian deflection (0”.87), which is, as it is known, two times smaller than relativistic deflection (Δ).

II.6 The advance of perihelion

The two prime integrals written as:

$$r\frac{dr}{dt} = \pm\sqrt{2hr^2 + 2\mu r - C}$$

and

$$\frac{d\theta}{dr} = \pm\frac{C - \omega r^2}{r\sqrt{2hr^2 + 2\mu r - C}}$$

$$h < 0,\ r = a(1 - e\cos u),\ C^2 = \mu a(1 - e^2),\ -2h = \frac{\mu}{a}$$

We find, after a simple reasoning:

$$d\theta = \frac{C}{\sqrt{\mu a}(1 - e\cos u)}du - \frac{\omega a^2(1 - e\cos u)^2 ae\sin u}{\sqrt{\mu a}(1 - e\cos u)ae\sin u} = \ldots$$

$$= \ldots - \sqrt{\frac{a^3}{\mu}}\omega(1 - e\cos u)du = \omega dt$$

From Kepler’s equation:

$$(1 - e\cosh u)du = \sqrt{\frac{\mu}{a^3}}dt$$

the advance will be:

$$\delta\omega = \int_0^{2\pi}\frac{C}{\sqrt{\mu a}(1 - e\cos u)}du - \omega T - 2\pi = -\omega T\ ,$$

Where T is the period and:

$$\int_0^{2\pi}\frac{C}{\sqrt{\mu a}(1 - e\cos u)}du = \sqrt{1 - e^2}\int_0^{2\pi}\frac{du}{1 - e\cos u} = 2\pi$$

as we find in [3].

We find something else, [4], if we observe that:

$$C - \omega r^2 = C_0$$

We have, after a variable changing and similar calculi as in inertial reference frames case:

$$d\theta = \frac{C_0}{\sqrt{\mu a}(1 - e\cos u)}du \Rightarrow$$

$$\Rightarrow \delta\omega = \int_0^{2\pi} \frac{C_0}{\sqrt{\mu a}(1-e\cos u)} du - 2\pi = \sqrt{1-e_0^2}\int_0^{2\pi}\frac{du}{1-e\cos u} - 2\pi = \frac{\sqrt{1-e_0^2}}{\sqrt{1-e^2}}2\pi - 2\pi =$$

$$= \left(\frac{C_0}{C} - 1\right)2\pi = -\frac{C_1}{C_0 + C_1}2\pi = -2\pi\frac{\omega}{\dot{\theta}+\omega}$$

Now if we are taking for the motion of our galaxy ω=-0”.0068/year and for the rotation of Earth around the Sun dθ/dt=360 degrees/year we find that the advance of perihelion is far from reality (5”.9 from observational data). Unfortunately we have the same situation with other planets (Mercury: 43”.11, Venus: 8”.4) because the advance of perihelion calculate with the above formula is very small, negligible.

Even we consider valuable the result last result the theoretical values for the advance of perihelion are not comparable with the observational data.

The theoretical values calculated with general relativity are: for Mercury, 43”, for Venus, 8”.6 and for Earth, 3”.8.

II.7 Eötvös effect

We have:

$$a_\rho = \ddot{r} - r(\dot{\theta}+\omega)^2 \quad \text{şi} \quad a_\eta = 2\dot{r}(\dot{\theta}+\omega) + r(\ddot{\theta}+\dot{\omega}).$$

The projections of forces acting on point mass P will look, after simplification and simple calculi:

$$a_\rho = -\frac{\mu}{r^2} - \ddot{r} + r\omega^2 + 2r\omega\dot{\theta} + r\dot{\theta}^2$$

$$a_\eta = -r\ddot{\theta} - 2\dot{r}\dot{\theta} - r\dot{\omega} - 2\dot{r}\omega$$

which are, without question, the same coordinates projections.

This observation is very important because it helps us to imagine a way to find out the correct form of the equations of motion in spherical coordinates in a simpler manner. The existence of previous section is justified not only by the deduction from other hypothesis than centrifugal force variation of Eötvös effect but to prove that the above equations are correct, [5]. The manner in which ω was included in these equations can be extrapolated for the correspondent equations in spherical coordinates. Consequently we have for the radial component of acceleration:

$$a_r = \ddot{r} - r\dot{\theta}^2 - r(\dot{\phi}+\omega)^2\sin^2\theta$$

The projected correspondent force acting on point mass P, after elementary calculi will be:

$$a_r = -\frac{\mu}{r^2} - \ddot{r} + r\dot{\theta}^2 + r\omega^2\sin^2\theta + r\dot{\phi}^2\sin^2\theta + 2r\omega\dot{\phi}\sin^2\theta$$

Assume that radial velocity of the motion is constant; therefore its derivative will be null. If ω is the angular velocity of the Earth then we can neglect also the correspondent centrifugal force.

It remain the non-null terms due to non-inertial motion and the gravitational acceleration If we keep only the terms of interest then we can write:

$$a_E = r\dot{\theta}^2 + r\dot{\phi}^2 \sin^2\theta + 2r\omega\dot{\phi}\sin^2\theta$$

If we are taking into account that, for horizontal velocity of the body on the Earth's surface, its vertical velocity and the relation between altitude and elevation angle, the expressions are:

$$u = r\dot{\phi}\sin\theta$$

$$v = r\dot{\theta}$$

$$\Phi = 90 - \theta$$

then we find an expression similar to expression found by Eötvös, in which the symbols have the same significations:

$$a_E = 2\omega u\cos\Phi + \frac{u^2 + v^2}{r}$$

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Chapter III
The study of eliptical movement in different formalisms

III.3 The spatial eleptical movement

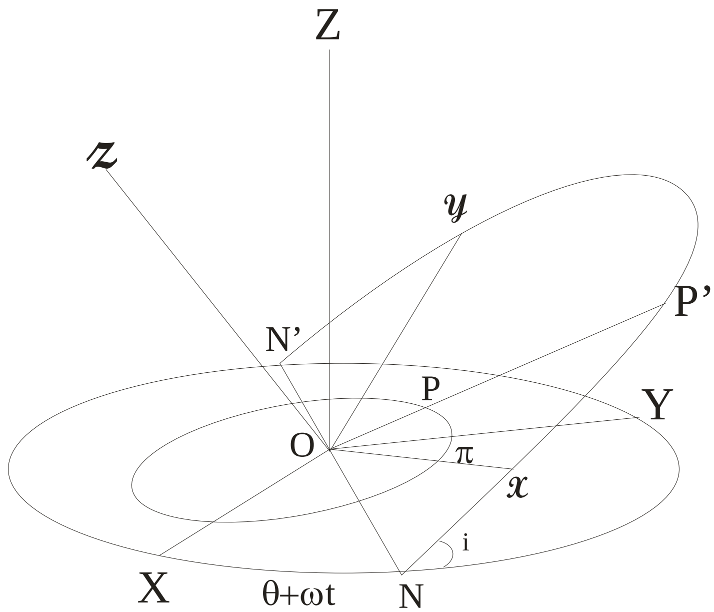

fig.III.1-the schematic representation of spatial movement parameters: $XN + Nx = \varpi$ is the perihelion longitude, $Nx = \varpi - \theta'$, $\theta' = \theta + \omega t = XN$ is node longitude, i the slope, τ perihelion passing time.

The six equations of perturbated movement are:

$$\left.\begin{aligned}
\frac{da}{dt} &= -\frac{2}{n^2}\frac{\partial R}{\partial \tau} \\
\frac{de}{dt} &= -\frac{\sqrt{1-e^2}}{na^2 e}\frac{\partial R}{\partial \varpi} - \frac{1-e^2}{n^2 a^2 e}\frac{\partial R}{\partial \tau} \\
\frac{di}{dt} &= -\frac{1}{na^2\sqrt{1-e^2}\sin i}\frac{\partial R}{\partial \theta'} - \frac{\tan 1/2}{na^2\sqrt{1-e^2}}\frac{\partial R}{\partial \omega} \\
\frac{d\theta'}{dt} &= \frac{1}{na^2\sqrt{1-e^2}\sin i}\frac{\partial R}{\partial i} \\
\frac{d\varpi}{dt} &= \frac{\sqrt{1-e^2}}{na^2 e}\frac{\partial R}{\partial e} + \frac{\tan 1/2}{na^2\sqrt{1-e^2}}\frac{\partial R}{\partial i} \\
\frac{d\tau}{dt} &= \frac{2}{n^2 a}\frac{\partial R}{\partial a} + \frac{1-e^2}{n^2 a^2 e}\frac{\partial R}{\partial e}
\end{aligned}\right\} \quad (a-f)$$

where we note $n = \sqrt{\frac{\mu}{a^3}}$.

Let's calculate now, as an example, the advance of perihelion.We have, hance:

$$\theta' = \theta + \omega t \qquad F = -\frac{\mu}{r^3} + \frac{C}{r^4} \quad R = \frac{\mu}{2r^2} - \frac{C}{4r^3}$$

where:

F – is the gravitational force that occurs in non-inertial frames of reference
R – potential function from which we derive the above force.

From (e) we have:

$$\frac{d\varpi}{dt}=\frac{\sqrt{1-e^2}}{na^2e}\frac{\partial R}{\partial e}+\frac{\tan i/2}{na^2\sqrt{1-e^2}}\frac{\partial R}{\partial i}=\frac{\sqrt{1-e^2}}{na^2e}F\frac{\partial r}{\partial e}+\frac{\tan i/2}{\sin i\left(\dot{\theta}+\omega\right)}$$

where it was considered equation (d). The derivative $\partial r/\partial e$ it will result from expression $r=a(1-e\cos u)$:

$$\frac{\partial r}{\partial e}=a\left(-\cos u+e\sin u\frac{\partial u}{\partial e}\right)$$

from Kepler's equation we obtain:

$$u-e\sin u=u(t-\tau),\quad (u-e\cos u)\frac{\partial u}{\partial e}=\sin u$$

result replaced in $\partial r/\partial e$ it will lead at:

$$\frac{\partial r}{\partial e}=a\left(-\cos u+\frac{e\sin^2 u}{1-e\cos u}\right)=a\frac{-\cos u+e}{1-e\cos u}$$

The equation (e) will be:

$$\frac{\partial\varpi}{\partial t}=\frac{\sqrt{1-e^2}}{na^2e}\left[\frac{-\mu}{r^3}+\frac{C}{r^4}\right]a\frac{-\cos u+e}{1-e\cos u}+\frac{\tan i/2}{\sin i\left(\dot{\theta}+\omega\right)}$$

Taking into account the variable changing $t\rightarrow u$ we will have:

$$\frac{\partial\varpi}{\partial u}=\frac{\sqrt{1-e^2}}{na^2e}\left[\frac{-\mu}{r^3}+\frac{C}{r^4}\right]a\frac{-\cos u+e}{1-e\cos u}\frac{1-e\cos u}{n}+\frac{\tan i/2}{\sin i\left(\dot{\theta}+\omega\right)}\frac{1-e\cos u}{n}$$

meaning:

$$\frac{\partial\varpi}{\partial u}=\frac{\sqrt{1-e^2}}{ae}\frac{\cos u-e}{(1-e\cos u)^3}+\frac{\sqrt{1-e^2}}{nae}\frac{\cos u-e}{n}\frac{C}{r^4}+\frac{\tan i/2}{\sin i\left(\dot{\theta}+\omega\right)}-\frac{1-e\cos u}{n}$$

Taking into account:

$$\frac{r}{a}=1-e\cos u$$

and:

$$\frac{C}{r^4}=\frac{C_1\left(C_1+2C_0\right)}{r^4}=\frac{\omega^2r^4+2\omega\dot{\theta}r^4}{r^4}$$

we ahve:

$$\delta\overline{\omega}=I^1+I^2+I^3$$

where:

$$I^1=\int_0^{2\pi}\frac{\sqrt{1-e^2}}{ae}\frac{\cos u-e}{(1-e\cos u)^3}du=\int_0^{2\pi}\frac{\sqrt{1-e^2}}{ae}\frac{-(1-e\cos u)+1-e^2}{(1-e\cos u)^3}du$$

or:

$$I^1=\frac{\sqrt{1-e^2}}{ae^2}\left[-I_2+\left(1-e^2\right)I_3\right]$$

The integrals I_2 and I_3 will be calculated using the general formula:

$$I_n=\int_0^{2\pi}\frac{du}{(1-e\cos u)^n}=\int_{-\pi}^{\pi}\frac{du}{(1-e\cos u)^n}$$

To solve this integral we must do the succesive variable changings:

$$\tan\frac{u}{2}=z,\qquad u=2\arctan z\Rightarrow$$

$$z=\sqrt{\frac{1-e}{1+e}}\tan\frac{\varphi}{2}$$

after which we obtain a polynomial function, easier to integrate:

$$I_n=\frac{1}{\left(1-e^2\right)^{n-\frac{1}{2}}}\int_0^{2\pi}(1+e\cos\varphi)^{n-1}d\varphi\Rightarrow$$

from which:

$$I_1=\frac{2\pi}{\sqrt{1-e^2}},\qquad I_2=\frac{2\pi}{\left(1-e^2\right)^{3/2}},\qquad I_3=\frac{\left(2+e^2\right)\pi}{(1-e)^{5/2}}\Rightarrow$$

And the firs integral result will be:

$$I^1=\frac{\pi}{a\left(1-e^2\right)}$$

The other two integrals are more easy to solve, and they are:

$$I^2=\int_0^{2\pi}\frac{\sqrt{1-e^2}}{n^2ae}(-\cos u+e)\left[\omega\left(\omega+2\dot{\theta}\right)\right]du=\frac{a^2\sqrt{1-e^2}}{\mu}2\pi\left[\omega\left(\omega+2\dot{\theta}\right)\right]$$

$$I^3=\int_0^{2\pi}\frac{\tan i/2}{\sin i\left(\dot{\theta}+\omega\right)}\frac{1-e\cos u}{n}du=2\pi a\sqrt{\frac{a}{\mu}}\frac{\tan i/2}{\sin i\left(\dot{\theta}+\omega\right)}$$

and the advance of perihelion will be [3]

$$\delta\varpi=\frac{\pi}{a\left(1-e^2\right)}+\frac{2\pi a^2\sqrt{1-e^2}}{\mu}\left[\omega\left(\omega+2\dot{\theta}\right)\right]+2\pi a\sqrt{\frac{a}{\mu}}\frac{1}{\dot{\theta}+\omega}\frac{\tan i/2}{\sin i}$$

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Chapter IV
A Newtonian alternative theory for MOND

IV.3 The Newtonian equivalent of cosmological constant

A potential with a form close to (6) has been used by Milne to derive the first Friedmann equation from the energy integral:

$$H^2 = \frac{8\pi G}{3}\rho + \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3} \qquad (7)$$

where the term containing the cosmological constant has been introduced by postulating a so-called expansion force. Nevertheless if we observe that equation (4) includes the term $B_2 r^2$ which contains the Newtonian equivalent of a cosmological constant, one may reconsider Milne's derivation.

First thing we must do is to presume valid the potential (4). In other words to consider as valid the hypothesis that the interior region of the supposed homogenous spherical shell will not be an equipotent region. Consequently we have:

$$\Phi_{(r)} = \frac{B_1}{r} + B_2 r^2 \qquad (8)$$

which is the potential (4) with $B_3 = 0$, an operation which simplifies (4) without the associated force law being altered.

Then we have:

$$B_1 = GM,$$

where G is the Newton's gravitational constant and M is the entire mass within the sphere, a constant with respect to time,

$$M = \frac{4\pi}{3}\rho \cdot r^3$$

and ρ is the mass density. We chose $B_1 = GM = B$, to be in accordance with Milne's derivation. Thus we expand the thickness of the spherical shell in vicinity of its center, in order to have a good approximation between our spherical shell and a compact sphere. The constant B_2 is presumed positive, it correspond to a repulsive force.

The force applied on a particle of mass m in motion within the potential (8) is:

$$m(d^2 r / dt^2) = -\frac{GMm}{r^2} + B_2 r^2 m \qquad (8')$$

This equation will lead us, by integration, to an equation of form (7). Multiplying it with the first derivative of 2r one will observe very easily that the left term is the derivative with respect to time of square first derivative. The last right term is obvious the derivative with respect to time of the repulsive potential. To calculate the first right term we need the expression:

$$\rho r \frac{dr}{dt} = -\rho r^3 \frac{d}{dt}\left(\frac{1}{r}\right) = \frac{d}{dt}\left[\rho r^3 \frac{1}{r}\right]$$

which result by the fact that the entire mass within the sphere, M, is a constant with respect to time and this leads to:

$$r^3 \frac{d\rho}{dt} + 3\rho r^2 \frac{dr}{dt} = 0$$

After we integrate (8'), introducing the scale parameter and replacing the integration constant with another constant proportional with the ratio r/a it results, after an elementary calculus, the energy integral:

$$H^2 = \frac{8\pi G}{3}\rho + \frac{kc^2}{a^2} + 2B_2 \qquad (9)$$

with k a dimensionless constant given by:

$$k = \frac{2E}{mc^2}$$

There is a physical equivalence between the two equations, observed directly from similarities between expressions (7) and (9). The only significant difference between them is the fact that one is deduced naturally from equation (8), the other is deduced from a postulated so-called expansion force.

In conclusion if we neglect equation (7) and set:

$$2B_2 = \frac{\Lambda c^2}{3} \tag{10}$$

we have been found a Newtonian equivalent for the cosmological constant.

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Chapter V
The variation of gravity constant with time in the framework of the expanding Universe

V.2 The variations of gravity constant in a Newtonian Universe

To validate the solution (5) for the additional part of the potential (3), it is necessary that this potential to be a solution of the Poisson equation. Since the potential (3), with solution (5), is specific to a spherical shell universe, and not to a compact sphere one, it is necessary to do a trick. So we must extend the thickness of the spherical shell near to its center. Under these conditions, solving the Poisson equation is formal. To additional part of the gravitational potential it will corresponds the energy density of the vacuum or the dark energy density. Of course, the latter is the best solution. But has the disadvantage that it is hard verifiable in practice. This is why we propose another solution to the Poisson equation, which has the advantage that it can be easily verified in practice and serves our purpose. What is it? To the additional part of the gravitational potential it will not longer corresponds a density of dark energy but a gravitational constant of the form:

$$G^{'} = G + \frac{2B_2 r^3}{3M}$$

To create a basis of comparison between the literature values obtained for the variation of the gravitational constant presented above and our results we need to transform the previous formula. If we make the derivative with respect to time and divide the result to G we get:

$$(\frac{dG/dt}{G}) = \frac{2B_2 r^3 H}{3MG} \tag{6}$$

where it has been taken into account the Hubble's law:

$$\frac{dr}{dt} = Hr$$

Formula (6) can be simplified taking into account (5), the expression of the mass:

$$M = \frac{4\pi\rho r^3}{3}$$

and equation (4), where it was made the assumption that we are in a flat space to be consistent with current observational realities:

$$\frac{8\pi\rho G}{3} = H^2 - \frac{\Lambda c^2}{3}$$

After a simple calculation equation (6) gets a much simpler and easier to implement value:

$$\left(\frac{dG/dt}{G}\right) = \frac{2}{3}\frac{1}{\frac{1}{H} - \frac{H}{\Lambda c^2}} \tag{7}$$

If the value for the Hubble constant is in agreement with current observations, i.e. 70 km / s / MPs and the cosmological constant is $10^{-52} m^{-2}$, expression (7) has the value $1{,}11 \times 10^{-10} yr^{-1}$.

Consider the equation (4), the first Friedmann equation. We apply to this equation the condition written as:

$$GM / r^2 = \Lambda c^2 r / 3 \tag{8}$$

If we write the mass M depending on matter density, the condition (8) will appear in a slightly altered form:

$$4\pi G\rho = \Lambda c^2 \tag{8'}$$

By introducing this condition in equation (4), it will result after some elementary steps the Friedmann equation specific to this case:

$$H^2 = 4\pi G\rho \tag{9}$$

There is an equivalent relation to (9), which is obtained from equation (4) and condition (8), but the expression is in accordance to the cosmological constant. We obtain the formula:

$$H^2 = \Lambda c^2 \tag{10}$$

If in expression (7) is taken into account (10), we obtain after some elementary steps the formula:

$$(dG/dt)/G = H/3 \tag{11}$$

It is obvious that this formula is valid only if the relation (8) is valid. If we adopt a value for the Hubble constant to be in unanimous acceptance of the international scientific community, namely $2{,}29 \times 10^{-18} s^{-1}$, then we can evaluate the expression (11) as $7{,}25 \times 10^{-11} yr^{-1}$. A value, if we look compared to the experimental values presented in the previous section, much closer to the experimental measurements than we expected.
An important consequence of equations (9) and (10) occur if we evaluate the ratio between material density and critical density from which the universe is flat:

$$\Omega = \rho / \rho_c$$

Taking into account the mentioned equations we can calculate this ratio as:

$$\Omega = 4\pi G\rho / H^2 = \Lambda c^2 / H^2 = 1 \qquad (12)$$

a limit value, which it make us to conclude that this case, in accordance to the condition (8), corresponds to the case of a static universe. If we actually go a little further with reasoning and calculate the deceleration parameter proper to condition (8):

$$q = -\ddot{a}a / \dot{a}^2$$

we see that to make this assessment it is easier to evaluate the second Friedmann equation specific to this case. It is quite obvious that starting from the general form of the second Friedmann equation:

$$\dot{H} + H^2 = \ddot{a} / a = -(4\pi G / 3)(\rho + 3p / c^2) + \Lambda c^2 / 3$$

where it takes into account the condition (8 '), the intermediate result is obtained:

$$\dot{H} + H^2 = \ddot{a} / a = -4\pi G p / c^2$$

With this intermediate result we can immediately assess the deceleration parameter:

$$q = -(\ddot{a} / a)(a^2 / \dot{a}^2) = p / \rho c^2$$

Now if we consider the state equation of the cosmic fluid:

$$p = \omega \rho c^2$$

and an universe dominated by matter, ω=0, it is obvious that we have the image of a static universe. Using this last result we can answer the question: what kind of universe is described by equation (4)? The ratio between material density and critical density specific to equation (4) is:

$$\Omega' = 8\pi G\rho / 3H^2 \qquad (13)$$

The ratio of the expressions (12) and (13) can be calculated very simple and it is 3 / 2. Now knowing that the limit value of the expression (12) is one, obviously it can be inferred that the expression (13) is 2 / 3. So a subunit value, which tells us that the universe is expanding forever, in matter domination epoch. Of course this is a purely theoretical rough value, the difference from experimental values are reflected only in the absence of matter and energy in the universe.

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Chapter VI
Allowed mass spectrum for scalars on Einstein branes in five dimensions

VI.3 Scalar fields in the bulk

In a space-time described by the metric::

$$ds_5^2 = e^{2f(z)} ds_4^2 + (dz)^2$$

the real scalar field which generates gravitation is expressed by the following lagrangean:

$$\underline{L}[\Phi]=\frac{1}{2}\eta^{ab}\Phi_{|a}\Phi_{|b}+\frac{\mu^2\Phi^2}{2} \tag{VI.18}$$

which leads to the 5-dimensional Gordon equation:

$$\Phi_{,55}+4f_{,5}\Phi_{,5}+e^{-2f}[\Delta-\frac{\partial^2}{\partial t^2}]\Phi=\mu^2\Phi \tag{VI.19}$$

where Δ is Laplace-Beltrami operator in S^3,

$$\Delta=\frac{1}{\alpha^2}\left\{\frac{1}{\sin(2\Theta}\frac{\partial}{\partial\Theta}\left[\sin(2\Theta)\frac{\partial}{\partial\Theta}\right]+\frac{1}{\cos^2\Theta}\frac{\partial^2}{\partial\alpha^2}+\frac{1}{\sin^2\Theta}\frac{\partial^2}{\partial\beta^2}\right\} \tag{VI.20}$$

After variable separation:

$$\phi=F(\Theta,\alpha,\beta,t)\cdot Z(z), \tag{VI.21}$$

the Gordon equations (VI.19) splits into the following system of decoupled equations:

$$\text{(a)}\qquad [\Delta-\frac{\partial^2}{\partial t^2}]F-\frac{2}{3a^2}\frac{M^2}{k^2}F=0$$

$$\text{(b)}\qquad \frac{d^2Z}{dz^2}+4k\coth(kz+C)\frac{dZ}{dz}+[\frac{M^2}{\sinh^2(kz+C)}-\mu^2]Z=0 \tag{VI.22}$$

By introducing the notations:

$$w=kz+C,\eta^2=\frac{M^2}{k^2},\varepsilon^2=\frac{\mu^2}{k^2} \tag{VI.23}$$

and the new function $G(w)=\sinh^{3/2}(w)\cdot Z(w)$, equation (VI.22.b) turns into:

$$\frac{d^2G}{dw^2}+\coth(w)\frac{dG}{dw}-\left[\left(\varepsilon^2+\frac{15}{4}\right)+\frac{\frac{9}{4}-\eta^2}{\sinh^2(w)}\right]G=0 \tag{VI.24}$$

being of the same form as the one satisfied by the toroidal functions:

$$\frac{d^2u}{dw^2}+\coth(w)\frac{du}{dw}-\left(l^2-\frac{1}{4}+\frac{m_l^2}{\sinh^2(w)}\right)u=0$$

which are equivalent, after a coordinate transformation, with the associated Legendre functions, [13]. Thus, the solutions (VI.22 b) are:

$$Z_{(\omega)}=\sinh^{-3/2}\omega\cdot G_{(\omega)} \tag{VI.25}$$

with $G_{(\omega)}=\left\{P^{ml}_{l-1/2}(\cosh\omega),Q^{ml}_{l-1/2}(\cosh\omega)\right\}$, while quantization conditions:

$$\varepsilon^2+\frac{15}{4}=l^2-\frac{1}{4},\ \frac{9}{4}-\eta^2=m_l^2\ , \tag{VI.26}$$

lead to the following nontrivial mass spectrum, [9]:

$$\mu^2 = (l-2)(l+2)k^2 . \qquad \text{(VI.27)}$$

Obviously, this is affecting the scalar evolving in the brane, whose mass takes only the following values:

$$m_*^2 \equiv \frac{2}{3a^2}\eta^2 = \frac{1}{a^2}[\frac{3}{2} - \frac{2}{3}m_l^2] = \{0, \frac{5}{6a^2}, \frac{4}{3a^2}, \frac{3}{2a^2}\}, \qquad \text{(VI.28)}$$

for $m \in \left\{\pm\frac{3}{2}, \pm 1, \pm\frac{1}{2}, 0\right\}$.

VI.4 Scalar mass field on the brane

Let us come from the bulk into the brane, along constant space dimensions and derive the propagator of the scalar field described by the Gordon equation (VI.22 a) with the mass spectrum (VI.28).

For the orthonormal set of eigenvalues $\Lambda_{\vec{n}}(\theta, \alpha, \beta)$

$$\Delta\Lambda_{\vec{n}} + (\omega_{\vec{n}}^2 - m_*^2)\Lambda_{\vec{n}} = 0 \qquad \text{(VI.29)}$$

with Δ given by (VI.20), we apply the variable separation:

$$\Lambda_{\vec{n}}(\theta, \alpha, \beta) = \Theta(\theta)A(\alpha)B(\beta) \qquad \text{(VI.30)}$$

and get:

$$A(\alpha) = e^{im_1\alpha}, B(\beta) = e^{im_2\beta}, m_1, m_2 \in Z \qquad \text{(VI.31)}$$

and the following equation for Θ

$$\frac{1}{\sin(2\Theta)}\frac{d}{d\Theta}\left(\sin(2\Theta)\frac{d\tau}{d\Theta}\right) + \left[a^2(\omega_{\vec{n}}^2 - m_*^2) - \frac{m_1^2}{\cos^2\Theta} - \frac{m_2^2}{\sin^2\Theta}\right]\tau = 0 \qquad \text{(VI.32)}$$

With the change of variable ζ=cos(2Θ) and the new function:

$$\Theta(\theta) = (1+\xi)^{\frac{m_1}{2}}(1-\xi)^{\frac{m_2}{2}}U(\xi), \qquad \text{(VI.33)}$$

the equation (VI.32) leads to a more familiar one:

$$(1-\xi^2)\frac{d^2U}{d\xi^2}[(m_2 - m_1) + (m_2 + m_1 + 2)\xi]\frac{dU}{d\xi}$$ **Error! Bookmark not defined.**

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(VI.34)

$$+\frac{1}{4}[a^2(\omega_{\vec{n}}^2 - m_*^2) - (m_1 + m_2)(m_1 + m_2 + 2)]U = 0$$

satisfied by the Jacobi polynomials, [13]:

$$U(\xi) = P_n^{(m_2,m_1)}(\xi) \quad \text{(VI.35)}$$

where:

$$P_n^{(m_2,m_1)}(\xi) = \frac{(-1)^n}{2^n n!}(1+\xi)^{-m_1}(1-\xi)^{-m_2}$$

$$\times \frac{d^n}{d\xi^n}[(1+\xi)^{n+m_1}(1-\xi)^{n+m_2}] \quad \text{(VI.36)}$$

with the spectrum:

$$\omega_{\vec{n}}^2 = m_*^2 + \frac{4}{a^2}(n+\frac{m_1}{2}+\frac{m_2}{2})(n+\frac{m_1}{2}+\frac{m_2}{2}+1); n \in N \quad \text{(VI.37)}$$

By introducing the new quantum numbers:

$$m^{'} = \frac{1}{2}(m_1+m_2), m = \frac{1}{2}(m_1-m_2), j = n+m \quad \text{(VI.38)}$$

and the factor:

$$\left[\frac{(j+m^{'})!(j-m^{'})!}{(j+m)!(j-m)!}\right]$$

we set up with the following orthonormal complete set of solutions of Laplace-Beltrami operator on S^3,

$$\Lambda_{\vec{n}}(\theta,\alpha,\beta) = \sqrt{\frac{2j+1}{V}} e^{i(m^{'}-m)\beta} W_{m^{'}m}^{j}(2\Theta) e^{i(m^{'}+m)\alpha}$$

$$\equiv D_{m^{'}m}^{j}(\Theta,\alpha,\beta) \quad \text{(VI.39)}$$

where $V = 2\pi^2\alpha^2$ is the volume of the sphere and $W_{m^{'}m}^{j}(2\Theta)$ are the Wigner functions

$$W_{m'm}^{j}(2\Theta) = \left[\frac{(j+m')!(j-m')!}{(j+m)!(j-m)!}\right]^{1/2} (\cos\Theta)^{m'+m} (\sin\Theta)^{m'-m}$$

$$\times P_{j-m}^{(m^{'}-m,m^{'}+m)}(\cos(2\Theta)) \quad \text{(VI.40)}$$

As in the well-known quantum field theory formalism, we express the field operators in terms of the complete set of eigenfunctions as:

$$\phi(x) = \sum_j \sum_{m^{'}} \sum_m \left[\frac{a_+(j,m^{'},m)}{\sqrt{2\omega_{\vec{n}}}} e^{i\omega_{\vec{n}}t} + \frac{a_-(j,m^{'},m)}{\sqrt{2\omega_{\vec{n}}}} e^{-i\omega_{\vec{n}}t}\right] \cdot D_{m^{'}m}^{j}$$

(VI.41)

$$\overline{\phi}(x) = \sum_j \sum_{m^{'}} \sum_m \left[\frac{b_+(j,m^{'},m)}{\sqrt{2\omega_{\vec{n}}}} e^{i\omega_{\vec{n}}t} + \frac{b_-(j,m^{'},m)}{\sqrt{2\omega_{\vec{n}}}} e^{-i\omega_{\vec{n}}t}\right] \cdot \overline{D}_{m^{'}m}^{j}$$

where $b_+(\vec{n}) = \bar{a}_-(\vec{n})$, $b_-(\vec{n}) = \bar{a}_+(\vec{n})$ so that the propagator, defined as:

$$D^{(c)}_{m_0}(\bar{x}, x) = \left\langle 0 \middle| T[\bar{\Phi}(\bar{x})\Phi(x)] \middle| 0 \right\rangle$$

will be concretely given by:

$$D^{(c)}_{m_*}(\bar{x}, x) = -\sum_j \sum_{m'=-j}^{j} \sum_{m=-j}^{j} (2\omega_{\vec{n}})^{-1} \{\Theta(\bar{t} - t)\exp[-i\omega_{\vec{n}}(\bar{t} - t)]$$

$$+ \bar{\Theta}(t - \bar{t})\exp[-i\omega_{\vec{n}}(t - \bar{t})]\} D^{j}_{m'm}(\bar{\Theta}, \bar{\alpha}, \bar{\beta}) \cdot D^{j}_{m'm}(\Theta, \alpha, \beta) \qquad \text{(VI.42)}$$

Finally, the discrete energy spectrum (VI.37), with the notations (VI.38), reads:

$$\omega^2_{\vec{n}} = \frac{1}{a^2}[\frac{3}{2} - \frac{2}{3}m_l^2 + 4j(j+1)] \qquad \text{(VI.43)}$$

and one may notice that, for each quantum number *j*, we have four distinct $\omega_{\vec{n}}$ values, corresponding to the allowed scalar masses in the brane (VI.28), [14].

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Chapter VII
Multidimensionality or fractality?

VII.1 Morphogenesis of gravitational structures through a non-differentiablehydrodynamics approach

Taking into account the complexity of the phenomena implied in the morphogenesis of the gravitational systems, we assume that the dynamics of these systems imply the fractal structure of space [2, 10, 11, 24-26].

If such an assumption works, then the dynamics of the gravitational systems in a fractal space are described by the covariant derivative [27-29]:

$$\frac{\hat{d}}{dt} = \frac{\partial}{\partial t} + \left(\hat{\mathbf{V}} \cdot \nabla\right) - i\mathcal{D}\Delta, \mathcal{D} = D\left(dt\right)^{\left(\frac{2}{D_F}\right)-1} \qquad (1)$$

where $\hat{\mathbf{V}}$ is the complex speed field

$$\hat{\mathbf{V}} = \mathbf{V}_D - i\mathbf{V}_F \qquad (2)$$

Here $\mathbf{V}_D$ is the standard classical speed (differentiable speed), which is independent of scale resolution (*dt*), while the imaginary part, $\mathbf{V}_F$, is a new quantity arising from non-differentiability (the fractal speed),

which is resolution-dependent; D is a structure coefficient, characteristic to the fractal-non-fractal transition, scale resolution and fractal dimension D_F dependent, and Δ is the Laplace operator.
We note that the use of any Kolmogorov or Haussdorff definitions [24 – 26] can be accepted for fractal dimension, but once a certain definition is admitted, it should be used until the end of analyzed dynamics. Moreover, our operator given by Eq. (1) is more general that the one of Nottale from SR [2, 10, 11]. Indeed, for movements on fractal curves with $D_F = 2$ (compatible with Brownian type movements) the operator given by Eq. (1) takes the form from SRT [2, 10, 11]

$$\frac{\hat{d}}{dt} = \frac{\partial}{\partial t} + \left(\hat{\mathbf{V}} \cdot \nabla\right) - iD\Delta$$

Applying the operator given in Eq. (1) to the complex speed field given by Eq. (2) and accepting the Newton's second generalized principle [2, 10, 11], in the form

$$\frac{\hat{d}\hat{\mathbf{V}}}{dt} = -\nabla U$$

we obtain a Navier-Stokes-type Eq.:

$$\frac{\hat{d}\hat{\mathbf{V}}}{dt} = \frac{\partial \hat{\mathbf{V}}}{\partial t} + (\hat{\mathbf{V}} \cdot \nabla)\hat{\mathbf{V}} - i\mathrm{D}\,\Delta\hat{\mathbf{V}} + \nabla U = 0 \qquad (3)$$

where $U = GM_0 m_0 / r$ is the gravitational scalar potential with G the Newton's constant, M_0 the rest mass of the gravitational source, m_0 the rest mass of the test particle and r the source - test particle distance.
Equation (3) means that at any point of any non-differentiable path, the local acceleration term, $\partial_t \hat{\mathbf{V}}$, the non-linear (convective) term, $(\hat{\mathbf{V}} \cdot \nabla)\hat{\mathbf{V}}$, the dissipative term, $\Delta\hat{\mathbf{V}}$, and the force term, ∇U, make their balance. Therefore, in a fractal space the gravitational system can be assimilated with a "rheological" fluid with imaginary viscosity, $iD\left(dt\right)^{(2/D_F)-1}$, whose dynamics is described by the complex speed field $\hat{\mathbf{V}}$. Moreover, since $\hat{\mathbf{V}}$ is a fractal functions [2, 10, 27-29], and presents self-similarity properties, some important correspondences with the holographic gravity [30, 31] can be realized.
If the motions of the gravitational system are irrotational:

$$\nabla \times \hat{\mathbf{V}} = 0, \nabla \times \mathbf{V}_D = 0, \nabla \times \mathbf{V}_F = 0 \qquad (4)$$

we can choose $\hat{\mathbf{V}}$ of the form:

$$\hat{\mathbf{V}} = -2i\mathrm{D}\,\nabla \ln \psi$$

For $\psi = \sqrt{\rho} e^{iS}$, with $\sqrt{\rho}$ the amplitude and S the phase of ψ, the complex speed field given by Eq. (2) takes the explicit form:

$$\begin{aligned} \hat{\mathbf{V}} &= 2\mathrm{D}\,\nabla S - i\mathrm{D}\,\nabla \ln \rho \\ \mathbf{V}_D &= 2\mathrm{D}\,\nabla S \\ \mathbf{V}_F &= \mathrm{D}\,\nabla \ln \rho \end{aligned} \qquad (5)$$

By substituting Eq. (5) in Eq. (3) and separating the real and the imaginary parts, up to an arbitrary phase factor which may be set to zero by a suitable choice of the phase of ψ, we obtain:

$$m_0[\frac{\partial \mathbf{V}_D}{\partial t} + (\mathbf{V}_D \cdot \nabla)\mathbf{V}_D] = -\nabla(Q + U) \qquad (6)$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}_D) = 0 \qquad (7)$$

with Q the non-differentiable potential:

$$Q = -2m_0 \mathsf{D}^2 \frac{\Delta\sqrt{\rho}}{\sqrt{\rho}} = -\frac{m_0 \mathbf{V}_F^2}{2} - m_0 \mathsf{D} \cdot \nabla \cdot \mathbf{V}_F \qquad (8)$$

and m_0 the rest mass of the fluid "entity".

The non-differentiable potential given by Eq. (8) comes from the non-differentiability of the movement curves and has to be treated as a kinetic term, not as a potential term. Moreover, the non-differentiable potential Q can generate a viscosity stress type tensor. Indeed, in the form:

$$Q = -m_0 \mathsf{D}^2 \left[\frac{\nabla^2 \rho}{\rho} - \frac{1}{2}\left(\frac{\nabla \rho}{\rho}\right)^2 \right] \qquad (41)$$

the non-differentiable potential induces the symmetric tensor

$$\sigma_{ik} = m_0 \mathsf{D}^2 \rho \nabla_i \nabla_k \ln \rho = m_0 \mathsf{D}^2 \left[\nabla_i \nabla_k \rho - \frac{(\nabla_i \rho)(\nabla_k \rho)}{\rho} \right] \qquad (42)$$

The divergence of this tensor is equal to the non-differentiable force density associated with Q:

$$\nabla \cdot \sigma = -\rho \nabla Q \qquad (43)$$

The quantity σ can be identified with the viscosity stress type tensor of a Navier-Stokes type equation:

$$m_0 \rho \frac{d\mathbf{V}_D}{dt} = \nabla \cdot \sigma \qquad (44)$$

The momentum flux density type tensor is

$$\pi_{ik} = \rho \mathbf{V}_{Di} \mathbf{V}_{Dk} - \sigma_{ik} \qquad (45)$$

and ~~it~~ satisfies the momentum – flow type equation

$$m_0 \frac{\partial}{\partial t}(\rho \mathbf{V}_D) = -\nabla \cdot \pi \qquad (46)$$

In order to complete the analogy to classical fluid mechanics, we introduce the kinematical and dynamical type viscosities:

$$\upsilon = \frac{1}{2} \mathsf{D} \qquad (47)$$

$$\mu = \frac{1}{2} m_0 \rho \mathsf{D} \qquad (48)$$

The quantities ν and μ are formal viscosities, both of them being induced by the fractal scale. Then, the tensor σ_{ik} takes the usual form:

$$\sigma_{ik} = \mu \left(\frac{\partial V_{Fi}}{\partial x_k} + \frac{\partial V_{Fk}}{\partial x_i} \right) \qquad (49)$$

Particularly, if σ_{ik} is diagonal, $\sigma_{ik} = \sigma \delta_{ik}$, Eqs. (6) and (7) take the form:

$$\left[\frac{\partial \mathbf{V}_D}{\partial t} + (\mathbf{V}_D \cdot \nabla \mathbf{V}_D) \right] = -\frac{\nabla \sigma}{\rho} \qquad (50)$$

$$\frac{\partial\rho}{\partial t}+\nabla\cdot(\rho\mathbf{V}_D)=0 \tag{51}$$

Equations (50) and (51) define, formally, a classical type hydrodynamics.

Further, using these equations for a plane symmetry, we can suggest another solution for the morphogenesis of the gravitational structures, without any need for an inflationary phase.

Thus, considering a barotropic type fluid, $\sigma=\rho c^2$, where *c* is the speed of sound in the fluid, and introducing the normalized coordinates

$$\omega t=\tau, kx=\xi, ky=\eta, \frac{V_x k}{\omega}=V_\xi, \frac{V_y k}{\omega}=V_\eta, \frac{\rho}{\rho_0}=N, \frac{kc}{\omega}=1 \tag{52}$$

where ω, *k* and ρ_0 are critical parameters of the fluid, the Eqs. (50) and (51) become:

$$\frac{\partial}{\partial\tau}\left(NV_\xi\right)+\frac{\partial}{\partial\xi}\left(NV_\xi^2\right)+\frac{\partial}{\partial\eta}\left(NV_\xi V_\eta\right)=-\frac{\partial N}{\partial\xi} \tag{53}$$

$$\frac{\partial}{\partial\tau}\left(NV_\eta\right)+\frac{\partial}{\partial\xi}\left(NV_\xi V_\eta\right)+\frac{\partial}{\partial\eta}\left(NV_\eta^2\right)=-\frac{\partial N}{\partial\eta} \tag{54}$$

$$\frac{\partial N}{\partial\tau}+\frac{\partial}{\partial\xi}\left(NV_\xi\right)+\frac{\partial}{\partial\eta}\left(NV_\eta\right)=0 \tag{55}$$

For the numerical integration we shall impose the initial conditions

$$V_\xi\left(0,\xi,\eta\right)=0, V_\eta\left(0,\xi,\eta\right)=0, N\left(0,\xi,\eta\right)=1/5, 1\le\xi\le 2, 0\le\eta\le 1 \tag{56}$$

as well as the boundary conditions

$$\begin{aligned}
&V_\xi\left(\tau,1,\eta\right)=V_\xi\left(\tau,2,\eta\right)=0,\quad V_\eta\left(\tau,1,\eta\right)=V_\eta\left(\tau,2,\eta\right)=0\\
&V_\xi\left(\tau,\xi,0\right)=V_\xi\left(\tau,\xi,1\right)=0,\quad V_\eta\left(\tau,\xi,0\right)=V_\eta\left(\tau,\xi,1\right)=0\\
&N\left(\tau,1,\eta\right)=N\left(\tau,2,\eta\right)=1/5\\
&N\left(\tau,\xi,0\right)=\frac{1}{10}\exp\left[-\left(\frac{\tau-1/5}{1/5}\right)^2\right]\exp\left[-\left(\frac{\xi-3/2}{1/5}\right)^2\right]\\
&N\left(\tau,\xi,1\right)=1/5
\end{aligned} \tag{57}$$

The Eqs. system (53) – (55) with the initial conditions given by Eq. (56) and the boundary ones given by Eq. (57) was numerically resolved by using the finite differences [40] (implemented by means of the NDSolve function in Mathematica 8.0).

We present in Figures 2 a, b – 4 a, b the numerical solutions for the normalized density field $N\left(\xi,\eta\right)$ - Figs. 2 a, b, for the normalized velocity field $V_\xi(\xi,\eta)$ - Figs. 3 a, b and for the normalized velocity field $V_\eta(\xi,\eta)$ - Figs. 4 a, b, at the normalized time sequence $\tau=1/2$, both three-dimensional solutions (Figs. 2a-4a) and through contour curves, two-dimensional solutions (Figs. 2b-4b).

Inspection of these numerical solutions shows the following: i) the normalized density field is of soliton-package-type [41]. Such numerical solution can explain for example the mass distribution of planets in the inner and outer of our solar system; ii) the normalized speed field V_ξ is symmetric with respect to the symmetry axis of the spatio-temporal Gaussian (fig. 2b); iii) vortices and shock waves type are induced at the periphery of structure for the normalized speed field V_η (fig. 3 b). Therefore, the non-differentiability of the space at large scales involves a transformation of the equations of motion into those of a macroscopic non-differentiable hydrodynamic system. As a consequence, there is a tendency to form gravitational structures at any epoch: these gravitational structures are described by the probability density distributions given by the square of the modulus of the probability amplitudes, which are solutions of this non-differentiable hydrodynamic system. The non-differentiable approach is fundamentally different from the classical one. The loss of determinism of individual trajectories is compensated by determinism of gravitational structures. At each epoch, stationary solutions may correspond to the shape of the non-differentiable potential and the limiting and matching conditions. These gravitational structures also evolve (as given by the time-dependent non-differentiable hydrodynamic system) in correspondence with the evolution of the environment [2].

It is possible that the El Nabulsi version of General Relativity [42] gives a complete answer to the problems previously mentioned. Moreover there is a tendency to form gravitational structures at any epoch with no need for an inflationary phase given the fractal structure of space (for details see [43-45]). We note that the use of fractal physical quantities in the description of the motions does not imply an inflationary phase (for details see [2]).

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Chapter VIII
The generalized gravitational potential

The condition:

$$m_{(\alpha)}\Phi_{(r)} = \frac{V_n n\sigma\alpha^{n-2}}{2r}\int_{r-\alpha}^{r+\alpha}\beta\Phi_{(\beta)}d\beta$$

has the solutions:

$$\Phi_n(r) = \frac{G_n}{r} \tag{7}$$

with the equivalent mass:

$$m_{(\alpha)} = V_n n\sigma\alpha^{n-1}$$

and the corresponding Yukawa-like potentials:

$$\Phi_n^{1Y} = \frac{G_n e^{-\xi r}}{r},\ \Phi_n^{2Y} = \frac{G_n e^{\xi r}}{r} \tag{8}$$

with the equivalent mass:

$$m_{(\alpha)} = V_n n\sigma\alpha^{n-2}\frac{sh(\xi\alpha)}{\xi}$$

The general solution:

$$\Phi = \Phi_n + \Phi_n^{1Y} \tag{9}$$

with the equivalent mass:

$$m\Phi = \sum_i m_i\Phi_i \tag{10}$$

is the same as the one obtained in [11]. Nevertheless, [12] was stated that in a n-dimensional space $\Phi \approx 1/r^{n-2}$. To be in accordance with this statement we must modify the equation (6) as follows:

$$m_{(\alpha)}\Phi_{(r)} = \frac{V_n n\sigma\alpha^{n-2}}{2r^{n-2}}\int_{r-\alpha}^{r+\alpha}\beta^{n-2}\Phi_{(\beta)}d\beta \tag{11}$$

This equation has the solutions:

$$\Phi_n^{'} = \frac{G}{r^{n-2}} \tag{12}$$

and:

$$\Phi_n^{'1Y} = \frac{Ge^{-\xi r}}{r^{n-2}}, \ \Phi_n^{'2Y} = \frac{Ge^{\xi r}}{r^{n-2}} \qquad (13)$$

with the same equivalent masses as (7) and (8).

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Final conclusions

The testing of non-inertial frames of reference dynamics, which has been made by us in the first part of this thesis, on some „standard" examples concerning the theory of relativity, brings, once again, another confirmation of general relativity, written by A. Einstein (see chapters I, II and III). The results about perihelion advance, the deflection of light rays by static gravitational fields and the redshift of photons emited in strong gravitational fields, are very eloquent in this sense. A supplementary argument is the study of Eötvös effect, which is also, a strong characteristic of the thesis. Because the initial theory, belonging to Eötvös, is pronounced intuitive, we tried, in this thesis, a better mathematically fundamented theory, which is in a better accordance with the heuristic principle (see chapter II). In addition, in the second part of the thesis, we had elaborated, in chapter IV, a Newtonian extensive theory of the MOND (Modified Newtonian Dynamics) description, which is based on a generalized gravitational potential which allow to embed the dark matter action in the dynamics of galaxies. Nevertheless, our version, conceived, at least, as a complement to MOND theory, is based on physically reasonable causes and explains a large variety of experimental data concerning the movement of galaxies.

The post-Newtonian cosmological model, elaborated in chapter V, generates a „never ending Universe", in accelerated expansion, in which intergalactic and extragalactic radiii are increasing, accordingly with the actual observational data.

In chapter VI we have studied an extension of gravitation based on the evolution of scalars within an Einstein Universe part of a 5-dimensional space. We obtain the orthonormal complet set of Gordon equation solutions and the mass spectrum. This is including the the discrete values, allowed, of massive scalar on the brane, according with an exact solution of 5-dimensional Einstein-Gordon equations.

Chapter VII analizes the morphogenesis of some gravitational structures, under hypothesis that the dynamics of a test particle in the gravitational field takes place on the continuous but non-differentiable curves. The dynamics of such gravitational systems is, firstly, described by a Navier-Stokes-like equation for a complex speed field which characterize its rheologic (with memory) behaviour. Therefore, the movement separation at the interactions scales within tha dynamics equation implies a non-differentiable hydrodynamics model. Finally, this aproximation was applied to the one-body problem and to the two-body problem and, trough a numerical simulation, to the morphogenesis of some gravitational structures. Consequently, intragalactic scale quantization (Solar System) and extragalactic scale quantization (Tifft's effect) impose some modifications to the Newtonian gravitational forces. In the same time, there exist a tendency of forming gravitational structures at any epoch, without to take into consideration a inflationary phase.

Chapter VIII is an attempt to generalize the well-known expression of the gravitational potential for more than three dimensions. We have used the Sneddon-Thornhill vision of the Newtonian gravity theorem and, then, we verofy our results with the Poisson's equation. The comparison with other theories it suggest some restrictions, but our results, generally, are valid until the experimental data will invalidate them.

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International Conferences

(1) The 14th edition of the annual TIM International Physics Conference, TIM 2013, november 21st-24th 2013, Timişoara, România, with two poster presentations:
M. B. Răuţ, Dark matter and the dynamics of galaxies: A Newtonian approach;
M. Agop şi M. B. Răuţ, Morphogenesis of gravitational structures through a non-differentiable hydrodynamics approach.

National Conferences

(1) The 3rd National Conference of Applied Physics, CNFA 2008, november 21st-22th 2008, Iaşi, România, with a poster presentation:
M. B. Răuţ, The spatial elliptical movement in a non-inertial frame of reference.
(2) The 4th National Conference of Applied Physics, CNFA 2010, november 19th-20th 2010, Iaşi, România, with two poster presentations:
M. B. Răuţ, Time invariance of the fundamental physical constants;
M. B. Răuţ, Variations of the gravitational constant with time in the framework of the expanding universe.
(3) The National Conference on Theoretical Physics, NCTP 2010, 4th edition, june 23rd -25th 2010, Iaşi, România, with a poster presentation:
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